

Sheet 4.

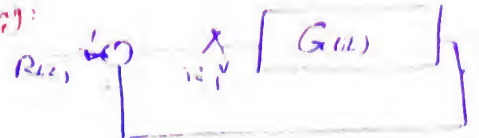
2 For the system shown in figure below with discrete transfer function

$$G(z) = \frac{0.1(z+0.9)}{(z-1)(z-0.7)}$$

Determine whether the system is stable or not using:

① Bilinear Transformation

② Jury Test



Solⁿ

$$C.L.T.F = \frac{C(z)}{R(z)} = \frac{G(z)}{1+G(z)} \Rightarrow \text{c/c eqn } 1+G(z)=0$$

$$\text{c/c eqn} = 1 + \frac{0.1(z+0.9)}{(z-1)(z-0.7)} = 0$$

$$(z-1)(z-0.7) + 0.1(z+0.9) = 0$$

$$z^2 - 1.6z + 0.79 = 0$$

① Bilinear Transformation,

$$z = \frac{1+r}{1-r}$$

$$\therefore R(z) = z^2 - 1.6z + 0.79 = 0 \quad \text{c/c eqn}$$

$$\left(\frac{1+r}{1-r}\right)^2 - 1.6\left(\frac{1+r}{1-r}\right) + 0.79 = 0$$

$$(1+r)^2 - 1.6(1+r)(1-r) + 0.79(1-r)^2 = 0$$

$$1 + 2r + r^2 - 1.6(1-r^2) + 0.79(1-2r+r^2) = 0$$

$$3.39r^2 + 0.42r + 0.19 = 0$$

Construct Routh Array

r^2	3.39	0.19
r	0.42	
r^0	0.19	

There is no sign change in first column of the array



Stable System

(2) Jury test

$$P(z) = z^2 - 1.6z + 0.79$$

- ① $P(1) = (1)^2 - 1.6(1) + 0.79 > 0$ ✓
- ② $P(-1) = (-1)^2 - 1.6(-1) + 0.79 > 0$ ✓
- ③ $|a_0| = 0.79 < |a_2| = 1$ ✓

Stable System

So For 2nd order system there is no need to find b_0, b_n and other elements.

③ Another Way (Poles Locations)

We can determine stability directly from Poles locations

- ① stable system if all poles inside the unit circle
- ② ~~Critical~~ Stable System if some poles on the Circumference of unit circle
- ③ Unstable System if one pole at least outside the unit circle

c/c eqn $z^2 - 1.6z + 0.79 = 0$

$$z_{1,2} = \frac{1.6 \pm \sqrt{(1.6)^2 - 4 \times 0.79}}{2} = 0.8 \pm j0.387$$

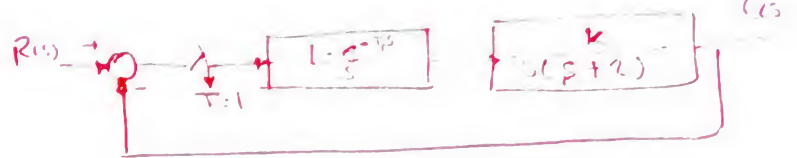
$$|z_{1,2}| = \sqrt{0.8^2 + 0.387^2} = 0.889 < 1$$

Stable System

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4 Find Range of k for stability

a) Solution



$$G(z) = \mathcal{Z} \left[\frac{1-e^{-Ts}}{s} \cdot \frac{k}{s(s+2)} \right]$$

$$= k(1-z^{-1}) \mathcal{Z} \left[\frac{1}{s^2(s+2)} \right] = k(1-z^{-1}) \mathcal{Z} \left[\frac{A_1}{s^2} + \frac{A_2}{s} + \frac{A_3}{s+2} \right]$$

$$A_1 = 0.5, A_3 = 0.25 \quad (\text{By inspection})$$

To find A_2

$$\text{let } s=1 \rightarrow \frac{1}{3} = 0.5 + \frac{0.25}{3} + A_2 \rightarrow A_2 = -0.25$$

$$G(z) = k \left(\frac{z-1}{z} \right) \mathcal{Z} \left[\frac{0.5}{s^2} - \frac{0.25}{s} + \frac{0.25}{s+2} \right] = k \frac{(z-1)}{z} \left[\frac{0.5z}{(z-1)^2} - \frac{0.25z}{z-1} + \frac{0.25z}{z-2} \right]$$

$$= k \left[\frac{0.5}{(z-1)} - 0.25 + \frac{0.25(z-1)}{z-0.135} \right]$$

$$= k \left[\frac{0.5(z-0.135) - 0.25(z-1)(z-0.135) + 0.25(z-1)^2}{(z-1)(z-0.135)} \right]$$

$$= k \left[\frac{0.5z - 0.0677 - 0.25(z^2 - 1.135z + 0.135) + 0.25(z^2 - 2z + 1)}{(z-1)(z-0.135)} \right]$$

$$= k \left[\frac{0.284z + 0.149}{(z-1)(z-0.135)} \right]$$

$$\text{c/c eqn} \rightarrow 1 + G(z) = 0$$

$$1 + \frac{k(0.284z + 0.149)}{(z-1)(z-0.135)} = 0$$

$$(z-1)(z-0.135) + k(0.284z + 0.149) = 0$$

$$z^2 + (0.284k - 1.135)z + (0.149k + 0.135) = 0$$

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Using Jury test :

For stable system

① $P(1) > 1$

$$\therefore 1 + 0.284k - 1.135 + 0.135 + 0.149k > 0 \rightarrow \boxed{k > 0}$$

② $P(-1) > 1$

$$\therefore 1 - 0.284k + 1.135 + 0.135 + 0.149k > 0$$

$$2.27 - 0.135k > 0$$

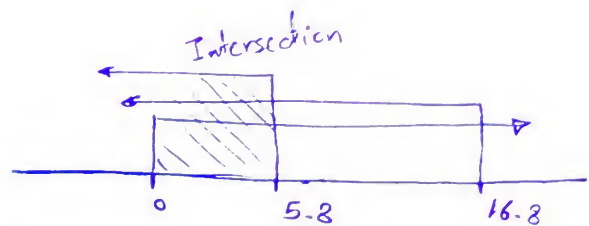
$$\boxed{k < 16.8}$$

③ $|a_0| < |a_1|$

$$0.135 + 0.149k < 1$$

$$0.149k < 0.865$$

$$\boxed{k < 5.8}$$

 \therefore The Range of k for stability $0 < k < 5.8$

b) solution

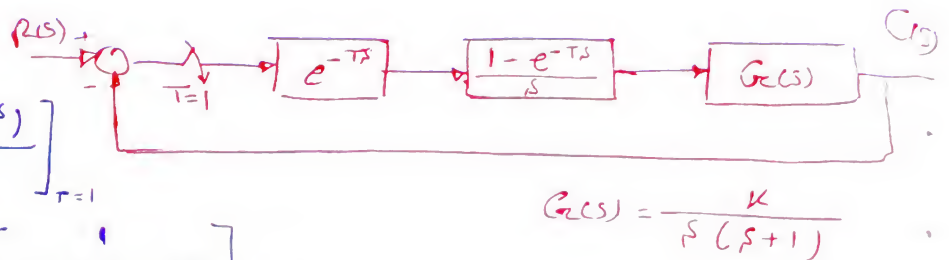
$$G(z) = \mathcal{Z} \left[\frac{k e^{-Ts} (1 - e^{-Ts})}{s^2 (s+1)} \right]_{T=1}$$

$$= k(1 - z^{-1}) z^{-1} \mathcal{Z} \left[\frac{1}{s^2 (s+1)} \right]$$

$$= \frac{k(z-1)}{z^2} \mathcal{Z} \left[\frac{1}{s^2} - \frac{1}{s} + \frac{1}{s+1} \right] = \frac{k(z-1)}{z^2} \left[\frac{z}{(z-1)^2} - \frac{z}{z-1} + \frac{z}{z-e^{-1}} \right]$$

$$= \frac{k}{z} \left[\frac{z}{z-1} - 1 + \frac{(z-1)}{z-0.368} \right] = \frac{k(z-0.368 - (z-1)(z-0.368) + (z-1)^2)}{z(z-1)(z-0.368)}$$

$$= \frac{k(z-0.368 - z^2 + 1.368z - 0.368 + z^2 - 2z + 1)}{z(z-1)(z-0.368)} = \frac{k(0.368z + 0.264)}{z(z-1)(z-0.368)}$$



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C/c eqn $1 + G(z) = 0$

$$1 + \frac{K(0.368z + 0.264)}{z(z-1)(z-0.368)} = 0$$

$$z(z-1)(z-0.368) + K(0.368z + 0.264) = 0$$

$$z^3 - 1.368z^2 + (1+K)(0.368z) + 0.264K = 0 \rightarrow \text{C/c eqn}$$

To check Stability for higher order systems use Bilinear Transformation $z = \frac{1+r}{1-r}$

$$\therefore \left(\frac{1+r}{1-r}\right)^3 - 1.368\left(\frac{1+r}{1-r}\right)^2 + 0.368(1+K)\left(\frac{1+r}{1-r}\right) + 0.264K = 0$$

$$(1+r)^3 - 1.368(1+r)^2(1-r) + 0.368(1+K)(1+r)(1-r)^2 + 0.264K(1-r)^3 = 0$$

$$1 + 3r + 3r^2 + r^3 - 1.368(1+r-r^2-r^3) + 0.368(K+1)(1-r-r^2+r^3) + 0.264K(1-3r+3r^2-r^3) = 0$$

$$(1+1.368+0.368+0.368K-0.264K)r^3 + (3+1.368-0.368-0.368K+0.792K)r^2 + (3-1.368-0.368K-0.368-0.792K)r + (1-1.368+0.368K+0.368+0.264K) = 0$$

$$(2.736+0.104K)r^3 + (4+0.424K)r^2 + (1.264-1.16K)r + 0.632K = 0$$

Routh Array

r^3	$2.736 + 0.104K$	$1.264 - 1.16K$
r^2	$4 + 0.424K$	$0.632K$
r^1	α	
r^0	$0.632K$	

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$$\Delta = \frac{(4+0.424K)(1.264-1.16K) - 0.632K(2-736+0.104K)}{4+0.424K}$$

$$= \frac{-0.558K^2 - 5.833K + 5.056}{4+0.424K}$$

For the System to be Stable
all elements of the 1st column in Routh array must be
Larger than zero (No Change in Sign)

$$\textcircled{1} \quad 2-736+0.104K > 0 \longrightarrow K > -26.31$$

$$\textcircled{2} \quad 4+0.424K > 0 \longrightarrow K > -9.434$$

$$\textcircled{3} \quad \Delta > 0$$

$$-0.558K^2 - 5.833K + 5.056 > 0$$

$$0.558K^2 + 5.833K - 5.056 < 0$$

$$K^2 + 10.45K - 9.061 < 0$$

$$\hookrightarrow \text{the roots of eqn } K_{1,2} = \frac{-10.45 \pm \sqrt{(10.45)^2 + 4(9.061)}}{2} = 0.805 \text{ \& } -11.26$$

$$(K - 0.805)(K + 11.26) < 0$$

$$K < 0.805$$

and

$$K > -11.26$$

$$K > 0.805$$

and

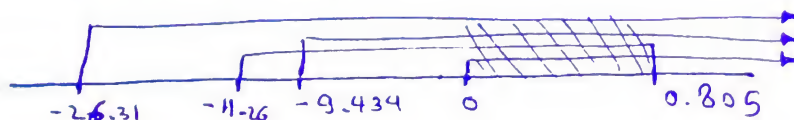
$$K < -11.26$$

illogical \rightarrow refused

$$\therefore -11.26 < K < 0.805$$

$$\textcircled{4} \quad K > 0$$

The Intersection Between all Conditions



the Range of K for stability

$$0 < K < 0.805$$